Modeling the Quantum Mechanical Measurement Process

Anton Amann[1]

The conceptual background of the quantum measurement problem is discussed with respect to an individual-stochastic interpretation of quantum mechanics in terms of pure states. The derivation of a stochastic dynamics on the pure state space of the "system to be measured" (starting from the joint system including an environment/measurement apparatus) is sketched. Finally the asymptotic behavior of such a derived stochastic dynamics is discussed: it is argued that usually one can*not* expect a measurement-type *asymptotic* behavior. Hence measurement-type behavior can only arise during an intermediate period of time and the asymptotic behavior will be thermal.

1. INTRODUCTION

My intention in this paper is to discuss the quantum mechanical measurement process from a conceptual point of view. Mathematical formulas and technicalities will be omitted as far as possible.

The most important conceptual point is a clear distinction between a *statistical interpretation* of quantum mechanics in terms of density operators (mixed states), on the one hand, and an *individual-stochastic interpretation* in terms of pure states, on the other. I will stick to an individual interpretation and shall give some reasons for this choice (see Section 2). Unfortunately, such an individual interpretation of quantum mechanics has not been worked out: In particular, it remains open what sort of (stochastic) dynamics (on the space of pure states of the system in question) is physically reasonable. I am convinced that stochastic dynamical equations should be *derived* from a "bigger" system, comprising not only the system to be measured, but also its "environment" (e.g., the measurement apparatus). A discussion of that

[1] Laboratory of Physical Chemistry, ETH-Zentrum, CH-8092 Zürich, Switzerland.

point will be given in Section 3. The last conceptual problem discussed here (see Section 4) concerns the asymptotic (large-time) behavior of such a derived stochastic dynamics: May one expect a measurement-type or much more a thermodynamic behavior?

2. STATISTICAL VERSUS INDIVIDUAL INTERPRETATIONS

To start with, consider an ammonia-type molecule with a ground-state vector Ψ_+ and a first-excited-state vector Ψ_- (it is permitted to think of a Born–Oppenheimer double-minimum potential). These two state-vectors are eigenvectors of the Hamiltonian, hence stationary. The respective energy difference is called $\Delta\epsilon$. In case of ammonia, the transition between these two eigenstates corresponds to the ammonia maser transition. By superposition, one may also prepare nonstationary states with state-vectors

$$\Psi_L := \frac{1}{\sqrt{2}} (\Psi_+ + \Psi_-)$$

$$\Psi_R := \frac{1}{\sqrt{2}} (\Psi_+ - \Psi_-)$$

These states "tunnel" back and forth, i.e., Ψ_L evolves dynamically into Ψ_R and back to Ψ_L, the frequency being that of the maser transition. The states corresponding to Ψ_L and Ψ_R have a pyramidal nuclear structure (whereas Ψ_+ and Ψ_- do not lead to a nuclear structure at all). Furthermore, Ψ_L is mapped into Ψ_R under space inversion and vice versa; hence they will be called left- and right-handed states, respectively (ammonia, NH_3, cannot have chiral nuclear structures, but NHDT has). The four states mentioned (with state-vectors Ψ_+, Ψ_-, Ψ_L, and Ψ_R) can be experimentally prepared in the case of ammonia (Kukolich et al., 1973). That is far from being trivial: Consider "properly" chiral molecules, such as a sugar or an amino acid, which are of ammonia type, too. There the proper ground state seems not to exist, or to be at least very unstable, decaying in a measurement-type process into the handed states (Amann, 1992; Hund, 1927; Pfeifer, 1980; Quack, 1986, 1989). The main difference between ammonia and, say, alanine concerns the energy difference $\Delta\epsilon$, which is about 9.5 J mol^{-1} for ammonia and around 10^{-60} J mol^{-1} for alanine (Pfeifer, 1980).

Now a measurement of the energy in an ammonia-type molecule is expected to end with the eigenstates Ψ_+ and Ψ_- in case of ammonia, and with the handed states Ψ_L and Ψ_R in case of alanine. Though these are completely different final results, the density operators of the respective mixtures, namely $\frac{1}{2}(D_+ + D_-)$ and $\frac{1}{2}(D_L + D_R)$ coincide. Hence a statistical description in terms of density operators will *not* distinguish between com-

pletely different situations. Nevertheless, one may expect that these states can be experimentally distinguished, since the states arising have different selection rules.

Consequently, the solution of a density-operator equation like

$$\frac{\partial}{\partial t} D_t = -\frac{\gamma}{4} [\sigma_3, [\sigma_3, D_t]] \tag{1}$$

can*not* straightforwardly be interpreted as a measurement-type process, even if the outer-diagonal elements of D_t vanish (with respect to a chosen basis) for large times t. Such density-operator dynamics can have completely different interpretations in terms of pure states (Wächter, 1991). Hence here I will replace density operators by *ensembles of pure states*, i.e., probability measures on the space of pure states of the system under investigation (Amann, 1994; Beltrametti and Bugajski, 1994; Misra, 1974).

The term "ensemble" is only used here faute de mieux. It is *not* generally thought to be a Gibbs ensemble. In a Gibbs ensemble of molecules there are no interactions or Einstein–Podolsky–Rosen correlations between the molecules; furthermore, it is not allowed to select a molecule from the ensemble and investigate it individually; all molecules of a Gibbs ensemble have to be treated in exactly the same way. The "ensemble" of ammonia molecules used above, on the other hand, is nothing but a collection of (possibly finitely many) states, with appropriate "probabilities." Also, in the ensemble of ammonia molecules used above the "temperature" is infinite; in such an ensemble transitions cannot be observed due to saturation. Nevertheless transitions can be observed when a single molecule of the ensemble is selected and investigated individually. If this single molecule is in the ground state, or has "temperature zero" and transitions to excited states can be observed. This whole discussion is, of course, heuristic, because certain assumptions concerning the behavior of individual molecules (à la Einstein–Bohr) are used. Incidentally, the difference between two ensembles of pure states with the same associated density operator concerns the fluctuations (around the mean value of an observable).

For the particularly simple spin-1/2 system, the state space is affinely isomorphic to the sphere in 3-space $\mathbb{R}^3$. Under this affine isomorphism the *pure* states (rays in 2-dimensional Hilbert space) correspond to the points on the *surface* S_2 of the sphere in 3-space. To any ensemble of pure states, i.e., to any probability measure μ on S_2, there corresponds a unique density operator D_μ, describing the respective mixture

$$\mathrm{Tr}(D_\mu \bullet) = \int_{S_2} \mathrm{Tr}(|\chi\rangle\langle\chi|\bullet)\, d\mu(|\chi\rangle\langle\chi|) \tag{2}$$

where $\text{Tr}(D_\mu\cdot)$ denotes the trace of the density operator D_μ multiplied with an arbitrary 2×2 matrix. Recall that the mixed state $\text{Tr}(D_\mu\cdot)$ cannot be uniquely decomposed into pure states: There exist many different ensembles μ leading to the same mixed state. That was precisely the reason to replace density operators by ensembles of pure states! It is perhaps worthwhile to mention that the support of the ensemble μ need *not* consist of orthogonal states: All sorts of mixtures are admitted.

A single individual system is described by a pure state [i.e., a state with a maximal set of truth-definite observables (Raggio, 1981)]. Ensembles of pure states describe the respective statistics. The *individual-stochastic interpretation of quantum mechanics* refers to such an ensemble theory. This interpretation of quantum mechanics is adapted to situations where single individual systems are investigated. Statistics may come in when similarly prepared systems are investigated (such as alanine molecules). This statistics refers to the individual states and not to expectation values. The usual "statistical interpretation" of quantum mechanics goes a step further by only admitting statistics of expectation values (instead of states).

3. DERIVING STOCHASTIC DYNAMICS

Let us consider the measurement of the 2×2 Pauli matrix σ_3. Starting from an arbitrary pure state $c_\alpha|\alpha\rangle + c_\beta|\beta\rangle$ (i.e., a point on the surface S_2 of the sphere), one would like to get to a final state which is either spin up or spin down (i.e., either the north or the south pole of the sphere). Figure 1 offers a visualization of that process.
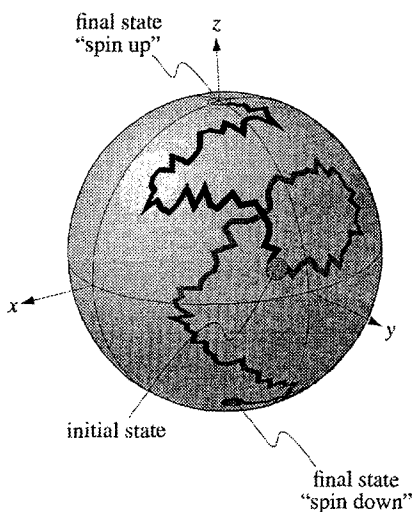

Fig. 1. The "quantum jump" during the measurement process is replaced by a stochastic dynamics leading to final states which are either spin up or spin down. This figure is *not* based on a simulation.

One might perhaps expect that the stochastic dynamics used would give rise to the final ensemble

$$\mu_f = |c_\alpha|^2 \delta_{\text{northpole}} + |c_\beta|^2 \delta_{\text{southpole}} \tag{3}$$

that is, would reproduce the "correct" transition probabilities. Nicely enough, there is a stochastic dynamics leading to this final ensemble. It is the following *Piron–Gisin process* (Gisin, 1984, 1989; Pearle, 1976) governed by the Stratonovich differential equation

$$d\vartheta(t) = -\sin\{\vartheta(t)\} \cos\{\vartheta(t)\} \ dt + \sin\{\vartheta(t)\} \circ dw(t) \tag{4}$$

where spherical coordinates have been used [$0 \le \vartheta \le \pi$; the azimuthal angle has been omitted; $w(t)$ is the Wiener process].

If one wants to "derive" this or other similar processes, one must be careful not to expect too much. A derivation usually starts from a larger system, incorporating the spin's environment (or measuring apparatus). One might, for example, think of a two-level molecule coupled to the quantum radiation field with an additional experimental input (e.g., proportional to a $\sin \omega t$ function). The dynamics of the joint system {molecule & field] will change an initial product state

$$(c_\alpha|\alpha\rangle + c_\beta|\beta\rangle) \otimes \Phi$$

(where Φ is the initial state of the field) into an entangled state

$$d_\alpha(|\alpha\rangle \otimes \Phi_\alpha) + d_\beta(|\beta\rangle \otimes \Phi_\beta)$$

Hence the molecule and the field are holistically correlated and no longer exist as individual systems (since the restriction of a pure nonproduct state to a subsystem is no longer a pure state). The hard problem with the quantum measurement process is indeed to show that one can get a product state back after an appropriate period of time or to show that a suitable dressing (i.e., a new tensorization) allows product states to survive (approximately) during time evolution. Perhaps it might also be possible to *derive* classical observables of the field [in the sense of Amann (1991)] and show that these classical observables *imply* a factorization of the states considered. At present only partial results are available (Gea-Banacloche, 1990, 1991). Let me stress again that a full quantum mechanical description of the joint system {molecule & environment} would, of course, never lead to a stochastic time evolution on the pure states of the two-level molecule (apart from rather trivial cases)! Therefore any stochastic time evolution of the two-level system's pure states is based on a factorization, which is imposed by hand! Hence, whatever complicated mathematical techniques (algebraic quantum mechanics, quan-

tum stochastic calculus, etc.) are used to get to a stochastic evolution equation—never trust that this factorization problem has been solved!

Presently it is therefore the best and most honest strategy to *assume* from the very beginning that only product states are admitted! Even with this *Hartree approximation* (which is used for the theory of spectroscopy without thinking about it) one can get a better understanding of the measurement process. And even with this Hartree approximation it is still difficult enough to understand it.

Hence the strategy is clear (Primas, 1990a–c; Zaoral, 1991): Starting with a Hamiltonian for the joint system {system to be measured & environment}, one writes down the corresponding Heisenberg differential equations, integrates them (the Hartree approximation being of enormous help), and replaces the quantum environment by a classical one (think of a high-temperature limit, and take the symmetrized quantum correlation function over to the classical environment). This procedure gives rise to a stochastic dynamics on the pure state space of the system to be measured (the two-level system in our discussion). The stochastic character of the dynamics can be traced back to the fact that the initial pure state of the environment (having infinitely many degrees of freedom) can*not* be experimentally determined. Hence only an appropriate guess is available, usually a *Gibbs distribution* with a certain temperature T.

4. RESULTS AND PROBLEMS

What are the results of this strategy? Well: Rather complicated stochastic diffusion equations (Primas, 1990a–c; Zaoral, 1991), which are not easily accessible to discussion. There are, in particular, some remarks to be made:

- There is absolutely no guarantee that the density operator evolution associated to such a stochastic diffusion equation via equation (2) fulfills a "closed" equation of the type

$$\dot{D}_t = f(D_t)$$

  as, for example, given in equation (1).

- Since we are dealing with nonlinear dynamical processes on the pure state space, there is no guarantee that the dynamics on the pure state space can be extended to all mixed states. But it can, of course, be extended to ensembles of pure states in the sense of Section 2.

- Starting from a given initial state $c_\alpha |\alpha\rangle + c_\beta |\beta\rangle$, one cannot hope that the transition probabilities to get to spin-up or spin-down states are precisely $|c_\alpha|^2$ and $|c_\beta|^2$, respectively. Consider, for example, the stochastic diffusion equations of the Piron–Gisin type

$$d\vartheta(t) = -2\kappa \, \sin\{\vartheta(t)\} \, \cos\{\vartheta(t)\}dt + 2\mu \, \sin\{\vartheta(t)\} \circ dw(t) \qquad (5)$$

with κ and μ being positive constants. As long as $\alpha := -\frac{1}{2}\kappa/\mu^2$ takes the same value $\alpha = -1$ as in the original Piron–Gisin process (with $\kappa = \mu = 1/2$), one gets the same transition probabilities, namely $\frac{1}{2}(1 \pm \cos\vartheta)$. For other values of the parameter α the transition probabilities change. With $\alpha = -2$, for example, the transition probabilities (for getting near the poles) are $\frac{1}{4}(2 \pm 3\cos\vartheta \mp \cos^3\vartheta)$, differing from the "correct" values by less than 0.1. For other values of α (in particular, for $|\alpha|$ small), the transition probabilities can differ from the correct values considerably (soon the difference is 50% of the correct value).

These problems are perhaps inconvenient, but there is no physical reason to expect simpler structures (such as semigroups on the density operator level). Nevertheless there are other points which lead us beyond inconveniences, perhaps even into trouble: Consider again the dynamics specified by equation (5). This stochastic dynamics can be "derived" by the strategy sketched at the end of Section 3 (at least if some uncontrollable "approximations" are used: it is not meant to describe the measurement process). The tricky point is that the constant μ depends on the temperature of the original Gibbs distribution (of the environment's pure states; see end of the last section). In order to get a process exhibiting the correct transition probabilities (i.e., $\alpha = -1$), it would be necessary to adjust the temperature of the Gibbs distribution. That is certainly not what one would like to have.

There is an even more disturbing problem concerning the asymptotic behavior of derived stochastic diffusion equations. Let me formulate two arguments:

Argument 1. The original probability distribution of the environment's pure states is assumed to be thermal, i.e., a Gibbs distribution with a certain temperature T. Hence also the final ensemble μ_f of the spin system's pure states is thermal and depends in particular on the temperature T of the field. Since a "small system" admits only *one* Gibbs distribution for a given temperature (symmetry breaking does not arise), the final ensemble $\mu_f = \mu_T$ does not depend on the initial state $c_\alpha|\alpha\rangle + c_\beta|\beta\rangle$. Therefore the asymptotic behavior of the respective derived stochastic dynamics does not reproduce the correct transition probabilities (in mathematical terminology such a behavior is called ergodic).

Argument 2. Consider a stochastic differential equation for the "system to be measured" which is derived from a dynamics for the joint system including the quantum environment (measurement apparatus). Then the coefficients of this stochastic differential equation cannot be easily controlled (apart from the actual input function, e.g., a sin-function, of the experimenter).

Consequently, (nonergodic) measurement-type behavior cannot be enforced by simple manipulation of parameters, and the "system to be measured" will have a tendency to behave in a thermal (ergodic) way. A typical example is given by the coupling to the quantum radiation field with terms $[\hat{P} - q\hat{A}]^2/2m$, where $\hat{A}$ is the transverse vector potential of the field. All the coupling constants of a derived spin-$\frac{1}{2}$-boson model (Pfeifer, 1980) are fixed, and hence the parameters in a derived stochastic differential equation cannot be tuned in an ad hoc way. Though these matters are delicate, the derived stochastic diffusion equation has a good chance to be ergodic.

How can the thermal and the measurement-process points of view be reconciled? Well, quite obviously, it is not necessary to have an *asymptotic* measurement-type behavior. It would be absolutely sufficient to have measurement-type behavior only during an intermediate period of time, such as between some nanoseconds and some days. Then the asymptotic behavior could still be thermal for large times. Seen from the Einstein–Bohr point of view, the spectroscopist induces transitions between different energy levels of the molecule (e.g., by giving a sin ωt input), and hence forces the molecule to forget about its original state. Similarly, the spontaneous transitions between different molecular energy levels show that one should *not* expect asymptotic measurement-type behavior: eigenstates of the molecular Hamiltonian (which are the final states in an energy measurement) have a limited lifetime, and hence also our stochastic dynamics should show a similar feature; i.e., if a given initial state travels to one of the eigenstates (regions around north and south poles in our example), there should always be a chance to leave such a region and travel to another eigenstate. It is rather plausible that the dynamics loses its memory during such a process (as in an ergodic situation), that is, a path that changed various times between different eigenstates will not remember its initial position. Incidentally, the example of spontaneous transitions also teaches us that the typical duration of stay in regions near the poles can also be rather short (recall Einstein's formula for spontaneous transition probabilities).

Let me conclude with two remarks:

Thermal and ergodic behavior were more or less identified in the above reasoning. Quite a few people would not be satisfied with that, asking at least for detailed balance in thermal equilibrium (as used, for example, by Einstein in the derivation of his formula for spontaneous transition probabilities). With this additional assumption one also gets increasing relative entropies

$$-\int_{S_2} f_t \log f_t \, d\mu_f$$

with respect to the final ensemble μ_f (here f_t is the Radon–Nikodym derivative of the ensemble at time t with respect to μ_f). It would be an interesting problem to investigate under what circumstances stochastic differential equations fulfilling this condition could be derived.

Arguing with spectroscopic examples, an "Einstein–Bohr point of view" was used, where (approximate) eigenstates of the Hamiltonian play an important role. I would like to understand its relation to the "von Neumann point of view," where eigenstates of the coupling operator (coupling the system to be measured to the measurement apparatus) are more important. It would be nice to have examples of different stochastic dynamics (on the pure state level) corresponding to these views, nevertheless leading to the same dynamics on the density operator level.

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